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WORKING PAPERS

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CAF DEVELOPMENT BANK OF LATIN AMERICA

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ABSTRACT

In an effort to increase female labor participation in Chile, firms with more than 19 women must pay for childcare for the children of their female employees. We evaluate the effects of such policy using a model that features firm heterogeneity and three factors of production: women, men and capital. In our model the policy misallocates resources, driving firms to stop hiring once they are close to the threshold, depressing female participation and wages. We calibrate the model to the Chilean economy, and analyze via counterfactuals the effects of removing this distortion. First, we find that the policy reduces female labor participation by 1%, and wages by 2%. Second, the policy redistributes welfare from men to women, increasing female welfare by 0.05%. Third, we suggest alternatives policies that would be more successful at increasing female labor participation. For example, financing childcare with a tax on profits would increase female labor participation by 4%, with similar welfare consequences as the policy in place.

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EL EFECTO DISTORSIONADOR DE LAS POLÍTICAS QUE DEPENDEN DEL TAMAÑO Y DE LOS FACTORES: EL PAPEL DE LA SUSTITUIBILIDAD DE LOS FACTORES EN LA MEDICIÓN DEL IMPACTO DE UNA POLÍTICA DE SUBSIDIOS PARA EL CUIDADO INFANTIL EN CHILE

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RESUMEN

En un esfuerzo de aumentar la participación laboral femenina en Chile, las firmas con más de 19 mujeres deben pagar por el cuidado de los niños de sus empleadas. Evaluamos los efectos de esa política con un modelo que introduce heterogeneidad de las firmas y tres factores de producción: mujeres, hombres y capital. En nuestro modelo la política desplaza recursos ya que las firmas dejan de contratar mujeres una vez que están cerca del umbral, lo que reduce la participación femenina y sus salarios. Calibramos el modelo para la economía chilena y analizamos a través de contrafactuales los efectos de eliminar esta distorsión. Primero, encontramos que la política reduce la participación laboral femenina por 1%, y los salarios por 2%. Segundo, la política redistribuye bienestar de los hombres a las mujeres, lo que aumenta el bienestar de las mujeres en 0,05%. Tercero, sugerimos políticas alternativas que serían más exitosas en aumentar la participación laboral femenina. Por ejemplo, financiar el cuidado de niños con un impuesto sobre los beneficios aumentaría la participación laboral femenina en un 4%, con consecuencias sobre el bienestar similares a las de la política vigente.

The distortionary effect of size- and factor-dependent policies: The role of factor substitutability in measuring the impact of a child-care subsidy policy in Chile*

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Abstract

In an effort to increase female labor participation in Chile, firms with more than 19 women must pay for childcare for the children of their female employees. We evaluate the effects of such policy using a model that features firm heterogeneity and three factors of production: women, men and capital. In our model the policy misallocates resources, driving firms to stop hiring once they are close to the threshold, depressing female participation and wages. We calibrate the model to the Chilean economy, and analyze via counterfactuals the effects of removing this distortion. First, we find that the policy reduces female labor participation by 1%, and wages by 2%. Second, the policy redistributes welfare from men to women, increasing female welfare by 0.05%. Third, we suggest alternatives policies that would be more successful at increasing female labor participation. For example, financing childcare with a tax on profits would increase female labor participation by 4%, with similar welfare consequences as the policy in place.

Keywords: Misallocation, Child-care, Female labor force

JEL Codes: E24, J13, J21

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1 Introduction

Women have traditionally borne most of the costs related to children. This has translated into women having lower labor force participation which can also otentially xplain a substantial fraction of the gender wage gap (see Myck and Paull, 2001; Blau and Kahn, 1997). Substantial efforts have been devoted by governments to alleviate the penalty for motherhood, from more generous maternity leave benefits to anti-discrimination laws. The current paper studies an alternative where a women's work is incentivized through a child-care subsidy. This has been shown to increase women's labor market participation in a context of a developed country where the subsidy is paid by general taxes (Baker et al., 2008). However, the policy we study in a developing country, Chile, is paid by the employer instead and only when that employer hires more than 19 women in their firm. While Prada et al. (2015) showed that part of the cost is paid by women employees, Escobar et al. (2016) also show that there is a substantial number of firms that avoid the law by restricting the number of female employees in their wagebill. The current paper thus attempts to quantify the aggregate costs of this policy in a general equilibrium framework and contrast it to alternative policies. We pay special attention to the effects on welfare, productivity, and gender inequality.

To do this, we elaborate a framework where firms use three types of inputs in their production function: male workers, female workers, and capital. We allow for each type of workers to have a different elasticity of substitution with capital, as Escobar et al. (2016) conclude that men and women have strong differences with respect to their elasticity to capital. We think this is very relevant to the discussion since the cost of restricting factor choices in front of a policy may be dramatically different depending on the capacity of factor substitution.[1] To see this, consider the extreme case where factors are perfect substitutes. In this case, the policy would not generate a productivity loss, since women can be easily substituted for capital or men. The productivity loss would be larger the harder it is to substitute women by another factor but the benefits of the policy in terms of labor supply of women would also be the greatest.

To model the supply of labor and capital, we consider a model of indivisible labor, as in Rogerson (1988) and Hansen (1985), since this model delivers predictions with regards to labor participation, one of the focuses of the policy we study. We expand this framework by assuming there are two types of labor: male and female. Also, we assume that some of these members have children, and must send them to a childcare facility if they decide to work.

As in Rogerson (1988), we assume markets are complete, which implies that we can model the agents as members of a single household, where labor lotteries determine who works. It also implies that, if having a child is the outcome of a random process, household members insure

[1] Erosa et al. (2010) study parental leave policies in the context of a general equilibrium model with a search-and-matching labor market. However their study does not incorporate other factors nor a notion of size distribution of firms.

against the possibility of having a child, and the entire household bears the cost.

Understanding how this policy may influence the labor force participation of women and the gender wage gap may be highly relevant in developing economies where formal female labor supply is still, in general, much below that of developed countries. In fact, Chile has one of the lowest female labor force in the OECD, around 55.8 percent in 2015, which is close to Italy and Korea and lower than the OECD average of 63.0 percent.

The policy has two different types of impact. First, it distorts the optimal allocation by discouraging the hiring of women, making some firms smaller relative to the optimal in terms of output, men, women, and capital. This generates welfare and productivity losses. The second impact of the policy is redistributive. While the policy may in principle benefit or hurt women, depending on whether the extra benefit they receive through the child-care subsidy is larger than the employment losses generated by the policy, we find that it redistributes welfare from men to women.

To assess the effect of the policy quantitatively, we calibrate our model to the Chilean economy. A key feature of the calibration is the childcare cost. We set this so that childcare costs, relative to GDP, is the same in the model as in the data. In the data, we estimate this to be close to 0.13% of GDP. This implies that the overall cost of this policy is relatively small, and therefore the impacts of this policy should be interpreted in this context.

The policy, compared to the first-best, reduces the welfare of men and increases it for women, but as mentioned above, since the program is relatively small, the magnitudes are relatively modest. However, the policy does have impact on women's wages and labor participation: compared to a world without any programs, the policy decreases women's wages by up to 2% and women's labor participation by up to 1%. The fact that we have more than one or two factors, as in other studies of misallocation, may in part explain why distributional effects are here more relevant than in studies where factor switching was less possible.

The welfare consequences of the policy we study are far from obvious, which is why providing quantitative estimates are so crucial. In this economy, the social planner compensates workers for their marginal product of labor. The childcare policy in Chile distorts this, and drives some firms to pay less than the marginal product of labor to women. In equilibrium, some firms that would hire close to 20 women absent the policy decide to cut down on women to save on childcare costs. Consequently, these firms pay female wages that are lower than the marginal productivity of women. In turn, they substitute women for men and capital, increasing the ratio of men to women and capital to women above the social planner's objective. Firms that hire more than 20 women are also distorted, since they pay for childcare in addition to wages, and this reduces their ratio of women to men and women to capital relative to firms not constrained by the policy.

We find that aggregate welfare, measured as the welfare of men plus the welfare of women, would increase, albeit very mildly, if the policy was abolished. In contrast, we find that the policy does indeed redistribute welfare from men to women. In units of consumption, it reduces welfare for men by 0.05%, and increases welfare of women by the same amount.

In sum, we find that the policy is not successful at increasing female labor participation, its main objective. While the policy in partial equilibrium would clearly increase female labor supply, the incentives it provides for firms to not hire women end up generating significantly less demand for female labor. In equilibrium, we find that wages fall slightly and female labor force participation falls. In light of this, we propose two alternative policies that would be more successful in increasing female labor participation. The first one uses a lump-sum tax to finance childcare. We find that this policy would increase female labor participation by almost 4%. Alternatively, the second policy finances childcare via corporate taxes, which are more realistic than lump sum ones. This would increase female labor participation by 4% as well.[2]

We find that the results are relatively robust to changing some of the parameters, in particular from the production function. We also show that were the policy to be expanded, for example, by 1 additional year of age or if mothers in Chile started working 10 percentage point more than actually, we would find that the effects of the policy would be more substantia,l which implies that part of the reason we see limited changes in some of the key variable of interest we have is because the policy is quite limited.

The costs of factor misallocation has been widely studied in the literature. Studies have shown that policies that generate misallocation of inputs lowers welfare and measured output per capita, as in Guner et al. (2008), Garicano et al. (2013), Gourio and Roys (2014), and Restuccia and Rogerson (2008), among others (see Restuccia and Rogerson, 2013, for a survey of the literature). In contrast, the substitutability between factors of production and how they interact with misallocation has been largely unexplored. Most papers include only labor, so that firms cannot substitute the source of misallocation for something else. At most, some of these papers include capital and labor, which allows for some degree of substitutability, but in this case, the substitutability is starker, since men and women are potentially more substitutable than capital and labor. We thus see our contribution as providing estimates that do take this substitution into account.

The rest of the paper is organized as follows. The next section presents the policy and its application in Chile. Section 3 then presents our theoretical framework, while the following section explains the calibration of the parameters. Section 7 then details our results and the last section concludes.

[2] A policy that would offer free childcare by any firm, irrespective of the number of women they hire would replicate the first-best scenario since firm size would not be distorted and firms would pass childcare costs entirely to their employees.

2 Policy

The policy we study in this paper was established at a moment where labor force participation of women was extremely low, in 1917. It was then denominated *Ley de Salas Cuna* and forced every factory, workshop or industrial establishment that hired 50 women or more (above age 18) to provide child care, specially conditioned to receive female employees' children under 1 year of age while the mother was working.[3] Over time, the law reduced the required number of women from 50 to 20 in 1931[4] and then later in 1987 it required the child care to be provided until the child was 2 years of age.[5] However, it remained, over the full period, a privileged reserved to the children of female employees: male employees who have children are unable to benefit from the subsidy. In 1998, the law also made it more difficult to evade as it prevented multi-plant firms to avoid being subject to the policy by having all their establishments below the required number of women, now including all plants in the calculations. As a result, the Chilean labor code in its article 203 currently says that all firms that hire 20 or more women must have annexed rooms where female workers can maintain and feed their children as long as they are below 2 years old. The code also indicates that the same obligation applies to commercial, industrial or services centers or complexes administered under a common legal entity which establishments hire in total 20 or more women.

In order to accomplish with the normative, firms have 3 options. First, they can create and maintain child-care centers annexed to the work place. Alternatively, firms can share child-care facilities with other establishments in the same geographic region. Finally, firms can also pay directly to external day care centers. In practice the latter is the most used modality and many firms simply offer a bonus to the mother, although it is not sufficient according to the law.

The cost of providing childcare for two years is relatively important for firms. According to Aedo (2007), the average cost of registering a child in a daycare was of CLP$100,000 per month (in 2002), which is about US$200. As a comparison, the average wage (for men and women) in the manufacturing sector in that same year was about CLP$222,000 per month. This suggests that this cost is relatively high compared to wage levels. A similar value is obtained by Rau (2010) who measures the cost of daycare for 2008 by calling 30 establishments and obtains an average value of CLP$137.438 for a full-day daycare. We use this to estimate that total childcare expenditure was about CLP$250 Billion in 2007, or about US$ 500 Million, which is about 0.13% of GDP. We provide the details of this estimate in section 6.

Escobar et al. (2016) estimate that in the Chilean manufacturing sector, there is a fall of around 20 percent in the number of firms at exactly 20 women. The fraction increases to 36 percent when looking at more recent periods where enforcement appears to have been stronger or when focus-

[3] Law 3.186, (1917), Chile.
[4] DFL 178, (1931), Chile.
[5] DL 2.200, (1987), Chile.

ing on firms with more than 100 employees. For sectors less intensive in women, the fall rises to about 40 percent, suggesting that a substantial number of firms are avoiding the legislation by remaining artificially small in women.

3 Model

Having explained the policy we will study, we now elaborate a general equilibrium model which will include the essential ingredients to capture how the policy may affect the economy.

This version of the model analyzes the different effects of the policy on men and women. There is one household in the economy, with a mass 1 of men and 1 of women. There is perfect insurance within each household, so that the cost of childcare and consumption is shared among all its members.

One of the main purposes of the policy studied is to affect female labor participation. For that reason, we model the decision to work as non-convex: either the household member works full time during period t, or does not work at all. Households can accumulate capital and open new firms.

Preferences for household member j are represented by the following utility function

$$\max \sum_{t=0}^{\infty} \beta^t [u(c_{jt}) - A_i h_{jt}]$$

where $i = male, female$, and $u'(c) > 0, u''(c) < 0$. A is a parameter that measures the disutility of working. We assume that this is the only difference between men and women: they each dislike working differently. The reason for this is that this provides flexibility to match the wage premium between men and women. Hours worked can be 0 or $\bar{h}$, that is $h_{jt} \in \{0, \bar{h}\}$.

All women have kids.[6] If a woman does not work, she takes care of her child, avoiding the childcare cost. But if she decides to work, the child must attend a childcare facility (or "sala cuna"). If the firm that employs the woman employs more than h^* women, the firm pays for the salacuna cost. Otherwise, the woman pays for it. Women do not know the number of women employees in firms, so they cannot look for firms that hire more than h^* women. In equilibrium, the probability of landing a job in a firm that pays for sala cunas at time t is d_t.

Note that in our model, a woman pays no cost for staying at home taking care of her child. She enjoys all the utility of not working while taking care of the child. This is relevant to understand the welfare results we will obtain although alternative versions would still imply something similar since this is calibrated to match the actual labor attachment of women in Chile.

[6]Since markets are complete, it would not make any difference to assume that only a fraction of them have kids, as long as labor lotteries are random over people with and without kids.

There are many firms with a technology that inputs men, women and capital to produce a homogeneous, representative output good. We follow Escobar et al. (2016) in modeling this technology:

$$F(\phi, k, h_f, h_m) = \phi \left(\left(k^\sigma + h_f^\sigma \right)^{\rho/\sigma} + h_m^\rho \right)^{\theta/\rho}$$

ϕ is a technology parameter that introduces heterogeneity into the problem. We assume that whenever a firm is born, the potential entrant draws its parameter ϕ from a distribution $G(\phi)$. Drawing this parameter, that is, starting a new firm, requires a fixed cost of κ units of the final output good. Firms die with an exogenous probability δ_f. $\theta \in (0, 1)$ implies decreasing returns to scale, so in an equilibrium under perfect competition, all firms produce a positive amount of output. Denote by M_t the mass of firms at time t. The timing for opening a new firm is as follows. If κ units of the final output good are invested in period t, a new firm with parameter ϕ will be alive in period $t+1$. Let N_t denote the mass of firms opened in period t. The law of motion for the mass of firms is

$$M_{t+1} = (1 - \delta_f) M_t + N_t$$

The production function is a nested CES function. We nest it this way so that the elasticity of substitution between capital and female labor can differ from that between capital and male labor. We show in the Appendix that the elasticity of substitution between capital and female labor is $(\sigma - 1)^{-1}$ and the elasticity of substitution between capital and male labor is $(\rho - 1)^{-1}$.

Capital accumulates according to $K_{t+1} = (1 - \delta_k) K_t + I_t$, where I_t is investment in time t, and it is in units of the final output good. Output equals total consumption plus investment in capital plus investment in new firms.

3.1 Equilibrium

We solve for an equilibrium with complete markets. There is a head of the household that designs consumption bundles for each member, and determines who needs to go to work. In equilibrium

- The head of the household determines how many men and women work each period
- They also decide on how much to consume and save as capital and firm creation
- Firms demand male labor, female labor, and capital from households
- Markets clear, that is, labor and capital supply equal to demand, and consumption plus investment equals output

- There is free entry. That is, the cost of opening a new firm cannot be less than the expected profits from such investment.

The Household

Rogerson (1988) shows that this solution can be decentralized also by making each household member make his or her own decisions, provided they take the probability of going to work as the price of the consumption good in a state in which they go to work, and one minus this probability as the price of the good when they do not.[7]

Without loss of generality, assume that households $j \in [0,1]$ are women and households $j \in (1,2]$ are men. The head of the household solves

$$\max \int_0^2 \left\{ \sum_{t=0}^{\infty} \beta^t [u(c_{jt}) - A_j h_{jt}] dj \right\}$$

$$s.t.$$

$$\sum_{t=0}^{\infty} p_t \left\{ \int_0^2 c_{jt} dj + I_t + \kappa N_t + \int_0^1 \mathcal{H}_{jt} \nu dj \right\} = \sum_{t=0}^{\infty} p_t \left\{ w_{ft} \int_0^1 h_j dj + w_{mt} \int_1^2 h_j dj + r_t K_t + \Pi_t \right\}$$

$$h_{jt} \in \{0, \bar{h}\}, \mathcal{H}_{jt} \in \{0,1\}, c_{jt} \geq 0, \ \forall j, t$$

$$I_t \geq 0, N_t \geq 0, \ \forall t \quad K_0 \text{ given}, M_0 \text{ given}$$

Here $\mathcal{H}_{jt}$ is an indicator function. If woman j works at time t, it is equal to 1. Otherwise, it is equal to 0. p_t is the price of the final good at time t, w_{ft} is the wage rate for a female worker at time t, and w_{mt} is the wage rate for a male worker at time t. r_t is the rental rate of capital at time t and Π_t are the profits of firms, which are part of the household's income. More specifically,

$$\Pi_t = \int_{-\infty}^{\infty} \pi_t(\phi) \mu_t(\phi) d\phi$$

where $\mu_t(\phi)$ is the measure of firms with productivity parameter ϕ at time t.

Acknowledging that all members of the same gender are symmetric, we can rewrite the

[7] A problem with this decentralization involves the distribution of K_0 and M_0 among individuals. The statement is true if all individuals have the same levels of K and M at time 0.

problem of the head of household as follows.

$$\max \sum_{t=0}^{\infty} \beta^t \left\{ \lambda_{ft}(u(c_{fw,t}) - A_f \bar{h}) + (1-\lambda_{ft})u(c_{fn,t}) + \lambda_{mt}(u(c_{mw,t}) - A_m \bar{h}) + (1-\lambda_{mt})u(c_{mn,t}) \right\} \tag{1}$$

$$\begin{aligned}
& s.t. \\
& \sum_{t=0}^{\infty} p_t \left\{ \lambda_{mt} c_{mw,t} + (1-\lambda_{mt}) c_{mn,t} + \lambda_{ft} c_{fw,t} + (1-\lambda_{ft}) c_{fn,t} + \right. \\
& \quad \left. K_{t+1} - (1-\delta_k)K_t + \kappa(M_{t+1} - (1-\delta_f)M_t) + \lambda_{ft}(1-d_t)\nu \right\} = \\
& \quad \sum_{t=0}^{\infty} p_t \left\{ \lambda_{ft} w_{ft} + \lambda_{mt} w_{mt} + r_t K_t + \Pi_t \right\} \\
& \quad \lambda_{ft} \in [0,1], \lambda_{mt} \in [0,1], c_{fwt} \geq 0, c_{fnt} \geq 0, c_{mwt} \geq 0, c_{mnt} \geq 0, K_t \geq 0, K_0 \text{ given.} \quad \forall i, t
\end{aligned}$$

where $\lambda \in [0,1]$ is the probability of working and the subscript f stands for female, m for male, w for worker, and n for non-worker. The maximization is with respect to the sequences $\{\lambda_{mt}, \lambda_{ft}, c_{mw,t}, c_{mn,t}, c_{fw,t}, c_{fn,t}, K_t\}$ and M_t.

Proposition 1 *In equilibrium, every household member has the same level of consumption. That is,* $c_{fwt} = c_{fnt} = c_{mwt} = c_{mnt}$.

Proof. The proof comes directly from the first order conditions. Let ω be the Lagrange multiplier. The first order conditions are:

$$\begin{aligned}
\lambda_{mt} &: \beta^t(u(c_{mw,t}) - A_m\bar{h} - u(c_{mn,t})) - \mu p_t(c_{mw,t} - c_{mn,t} - \nu) = -\omega p_t w_{m,t} \\
\lambda_{ft} &: \beta^t(u(c_{fw,t}) - A_f\bar{h} - u(c_{fn,t})) - \mu p_t(c_{fw,t} - c_{fn,t} - (1-d_t)\nu) = -\omega p_t w_{f,t} \\
c_{mw,t} &: \lambda_{m,t}\beta^t u'(c_{mw,t}) = \omega p_t \lambda_{m,t} \\
c_{mn,t} &: (1-\lambda_{m,t})\beta^t u'(c_{mn,t}) = \omega p_t (1-\lambda_{m,t}) \\
c_{fw,t} &: \lambda_{f,t}\beta^t u'(c_{fw,t}) = \omega p_t \lambda_{f,t} \\
c_{fn,t} &: (1-\lambda_{f,t})\beta^t u'(c_{fn,t}) = \omega p_t (1-\lambda_{f,t}) \\
k_t &: \frac{p_t}{p_{t+1}} = r_{t+1} + 1 - \delta_k \\
& \frac{u'(c_{fw,t})}{\beta u'(c_{fw,t+1})} = r_{t+1} + 1 - \delta_k
\end{aligned}$$

From these equations, it follows that $c_{mw,t} = c_{mn,t} = c_{fw,t} = c_{fn,t} = c_t$. ■

Entrant Firms

Potential entrants enter as long as the cost of entering is less than the expected profits from entering. A firm with parameter ϕ has period t profits equal to $\pi(\phi)$. The free entry condition states that the expected discounted profits cannot exceed the entry cost. Potential entrants that pay the entry cost κ draw a productivity parameter ϕ from a distribution $G(\phi)$. The free entry condition is

$$\kappa \geq \sum_{s=t}^{\infty}(1-\delta_f)^{s-t} q_{st} \int_{-\infty}^{+\infty} \pi_s(\phi) G(\phi) d\phi$$

where q_{st} is the discount factor between periods t and s, that is,

$$q_{st} = \prod_{v=s}^{t} \tilde{q}_v$$

where $\tilde{q}_v$ is the riskless discount rate in period v, which, by the no arbitrage condition, satisfies $\tilde{q}_v = \frac{1}{r_v + 1 - \delta_k}$.

Assuming positive entry, this condition holds with equal sign. We can rewrite this as

$$\begin{aligned}\kappa =& \sum_{s=t}^{\infty}(1-\delta_f)^{s-t} q_{st} \int_{-\infty}^{+\infty} \pi_s(\phi) G(\phi) d\phi = \\ & \tilde{q}_t \left(\int_{-\infty}^{+\infty} \pi_s(\phi) G(\phi) d\phi + (1-\delta_f) \sum_{s=t+1}^{\infty} (1-\delta_f)^{s-t} q_{s,t+1} \int_{-\infty}^{+\infty} \pi_s(\phi) G(\phi) d\phi \right)\end{aligned}$$

Thus,

$$\kappa = \tilde{q}_t \left(\int_{-\infty}^{+\infty} \pi_s(\phi) G(\phi) d\phi + (1-\delta_f)\kappa \right) \tag{2}$$

Incumbent Firms

Incumbent firms solve the following problem

$$\begin{aligned}\Pi_t(\phi) = \max\{ & \max_{\{h_m, h_f \leq \bar{h}, k\}} Y(\phi, k, h_f, h_m) - r_t k - w_{mt} h_m - w_{ft} h_f, \\ & \max_{\{h_m, h_f > \bar{h}, k\}} Y(\phi, k, h_f, h_m) - r_t k - w_{mt} h_m - (w_{ft} + \nu) h_f \}\end{aligned}$$

$\bar{h}$ is the maximum number of women that can be hired without needing to pay the cost of daycare.

Proposition 2 *In equilibrium, there exist thresholds ϕ_{1t} and ϕ_{2t}, with $\phi_{1t} < \phi_{2t}$ such that, if $\phi < \phi_{1t}$, a firm acts as if there was no childcare policy, if $\phi \in (\phi_{1t}, \phi_{2t})$, a firm hires $\bar{h}$ women, and if $\phi \geq \phi_{2t}$ a firm* *pays childcare costs to all its female workers.*

Proof. The proof works by comparing the profits of a constrained firm with those of an unconstrained one. Profits are maximized by computing the first order conditions. The first order conditions for a firm that hires less than or equal to $\bar{h}$ women are

$$\phi\theta\left[\left(k^{\sigma}+h_f^{\sigma}\right)^{\frac{\rho}{\sigma}}+h_m^{\rho}\right]^{\frac{\theta-\rho}{\rho}}\left(k^{\sigma}+h_f^{\sigma}\right)^{\frac{\rho-\sigma}{\sigma}}k^{\sigma-1}=r_t \tag{3}$$

$$\phi\theta\left[\left(k^{\sigma}+h_f^{\sigma}\right)^{\frac{\rho}{\sigma}}+h_m^{\rho}\right]^{\frac{\theta-\rho}{\rho}}\left(k^{\sigma}+h_f^{\sigma}\right)^{\frac{\rho-\sigma}{\sigma}}h_f^{\sigma-1}=w_{ft} \tag{4}$$

$$\phi\theta\left[\left(k^{\sigma}+h_f^{\sigma}\right)^{\frac{\rho}{\sigma}}+h_m^{\rho}\right]^{\frac{\theta-\rho}{\rho}}h_m^{\rho-1}=w_{mt} \tag{5}$$

Denote the profits of a firm with productivity parameter θ that hires its inputs by solving equations (3) through (5) by $\pi_{ut}(\phi)$, where the u stands for unconstrained.

A firm that hires more than $\bar{h}$ women has the following first order conditions:

$$\phi\theta\left[\left(k^{\sigma}+h_f^{\sigma}\right)^{\frac{\rho}{\sigma}}+h_m^{\rho}\right]^{\frac{\theta-\rho}{\rho}}\left(k^{\sigma}+h_f^{\sigma}\right)^{\frac{\rho-\sigma}{\sigma}}k^{\sigma-1}=r_t \tag{6}$$

$$\phi\theta\left[\left(k^{\sigma}+h_f^{\sigma}\right)^{\frac{\rho}{\sigma}}+h_m^{\rho}\right]^{\frac{\theta-\rho}{\rho}}\left(k^{\sigma}+h_f^{\sigma}\right)^{\frac{\rho-\sigma}{\sigma}}h_f^{\sigma-1}=w_{ft}+\nu \tag{7}$$

$$\phi\theta\left[\left(k^{\sigma}+h_f^{\sigma}\right)^{\frac{\rho}{\sigma}}+h_m^{\rho}\right]^{\frac{\theta-\rho}{\rho}}h_m^{\rho-1}=w_{mt} \tag{8}$$

Denote their profits by $\pi_{st}(\phi)$, where the s stands for "salacuna". In both of these cases, it is straightforward to show that female workers and profits are proportional to $\phi^{\frac{1}{1-\theta}}$, which is increasing in ϕ since $\theta < 1$. Thus, as ϕ increases, so do profits and female workers. If ϕ is small enough, then firms choose to hire less than $\bar{h}$ women, even without the policy in place. As ϕ increases, the first order conditions (3 through 5) require greater levels of h_f. By continuity, there exists some ϕ such that a firm with such level of ϕ chooses to hire $\bar{h}$ women. Denote that level by ϕ_{1t}.

A small increase in ϕ_t slightly increases the demand for women according to conditions (3) through (5), only that now the cost per female employee increased to $w_{ft}+\nu$. Thus, a firms with ϕ slightly larger than ϕ_{1t} will hire $\bar{h}$ women, and set capital and male employees by setting the following first order conditions:

$$\phi\theta\left[\left(k^{\sigma}+\bar{h}^{\sigma}\right)^{\frac{\rho}{\sigma}}+h_m^{\rho}\right]^{\frac{\theta-\rho}{\rho}}\left(k^{\sigma}+\bar{h}^{\sigma}\right)^{\frac{\rho-\sigma}{\sigma}}k^{\sigma-1}=r_t \tag{9}$$

$$\phi\theta\left[\left(k^{\sigma}+\bar{h}^{\sigma}\right)^{\frac{\rho}{\sigma}}+h_m^{\rho}\right]^{\frac{\theta-\rho}{\rho}}h_m^{\rho-1}=w_{mt} \tag{10}$$

Denote the profits of these firms by $\pi_{ct}(\phi)$. It is easy to show that there exists some $\epsilon > 0$ such that $\pi_{ct}(\phi_{1t}+\epsilon) > \pi_{st}(\phi_{1t}+\epsilon)$, so that a firm with $\phi_{1t}+\epsilon$ will hire $\bar{h}$ women. As ϕ increases, the difference between $pi_{ct}(\phi)$ and $\pi_{st}(\phi)$ drops, until it becomes negative. By continuity, there exists some level ϕ_{2t} such that $\pi_{ct}(\phi_{2t}) = \pi_{st}(\phi_{2t})$. For all $\phi > \phi_{2t}$, firms pay for childcare costs to women employees and use conditions (6) through (8) to hire men, women, and capital. ■

The existence of these three areas creates a misallocation of resources. More precisely, the marginal productivity of women will not be equal to their marginal cost for those firms with $\phi \in (\phi_1, \phi_2)$. This misallocation produces an efficiency loss. Moreover, it restricts demand for female labor. To see the misallocation of resources, the next two sections compute the undistorted equilibrium and the solution to the central planner problem for this economy, where each agent has the same weight on the social welfare function. They show that the central planner solution is the same as the decentralized solution with no distortion.

4 The Undistorted Equilibrium

The undistorted equilibrium has the household solving problem (1), just as in the distorted case. The firms, however, solve a different problem. This problem is

$$\Pi_t(\phi) = \max_{\{h_m,h_f,k\}} Y(\phi,k,h_f,h_m) - r_t k - w_{mt}h_m - w_{ft}h_f$$

with first order conditions given by equations (3) through (5). The free entry condition is the same as in the distorted case.

5 The Social Planner

The problem of the social planner is

$$
\begin{aligned}
&\max \sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 [u(c_{jt}) - A_f h_{jt}] dj + \int_1^2 [u(c_{jt}) - A_m h_{jt}] dj \right\} \\
s.t. &\int_0^2 c_{jt} dj + K_{t+1} - (1-\delta_k) K_t + \kappa (M_{t+1} - (1-\delta_f) M_t) + \nu \int_0^1 \mathcal{H}_{jt} dj = \\
&M_t \int_{-\infty}^{+\infty} \phi F(k_t(\phi), h_{mt}(\phi), h_{ft}(\phi)) G(\phi) d\phi \\
&K_t = M_t \int_{-\infty}^{+\infty} k(\phi) G(\phi) d\phi \\
&\int_0^1 h_{jt} dj = M_t \int_{-\infty}^{+\infty} h_{ft}(\phi) G(\phi) d\phi \\
&\int_1^2 h_{jt} dj = M_t \int_{-\infty}^{+\infty} h_{mt}(\phi) G(\phi) d\phi \\
&k_t(\phi) \geq 0, h_{mt}(\phi) \geq 0, h_{ft}(\phi) \geq 0 \;\; \forall t, \phi \\
&K_t \geq 0, h_{jmt} \in \{0,1\}, h_{jft} \in \{0,1\}, c_{jt} \geq 0 \;\; \forall j, t
\end{aligned}
$$

By a similar argument to that of Proposition 1, we can show that the social planner will assign the same level of consumption to all household members, and choose the probability that each man and each woman works. Thus, we can rewrite the problem as

$$
\begin{aligned}
&\max \sum_{t=0}^{\infty} \beta^t [2u(c_t) - A_m \lambda_{mt} - A_f \lambda_{ft}] \\
s.t.\; &2c_t + K_{t+1} - (1-\delta_k) K_t + \kappa (M_{t+1} - (1-\delta_f) M_t) + \nu \lambda_{ft} = \\
&M_t \int_{-\infty}^{+\infty} \phi F(k_t(\phi), h_{mt}(\phi), h_{ft}(\phi)) G(\phi) d\phi \\
&K_t = M_t \int_{-\infty}^{+\infty} k(\phi) G(\phi) d\phi \\
&\lambda_{mt} = M_t \int_{-\infty}^{+\infty} h_{mt}(\phi) G(\phi) d\phi \\
&\lambda_{ft} = M_t \int_{-\infty}^{+\infty} h_{ft}(\phi) G(\phi) d\phi \\
&k_t(\phi) \geq 0, h_{mt}(\phi) \geq 0, h_{ft}(\phi) \geq 0 \;\; \forall t, \phi \\
&K_t \geq 0, \lambda_{mt} \in [0,1], \lambda_{ft} \in [0,1], c_t \geq 0 \;\; \forall t
\end{aligned}
$$

Let $Y_t = M_t \int_{-\infty}^{+\infty} \phi F(k_t(\phi), h_{mt}(\phi), h_{ft}(\phi))G(\phi)d\phi$. Simplify this to

$$\max \sum_{t=0}^{\infty} \beta^t \left[2u \left(\frac{Y_t - (K_{t+1} - (1-\delta_k)K_t + \kappa(M_{t+1} - (1-\delta_f)M_t) + \nu M_t \int_{-\infty}^{+\infty} h_{ft}(\phi)G(\phi)d\phi)}{2} \right) \right.$$
$$\left. -A_m M_t \int_{-\infty}^{+\infty} h_{mt}(\phi)G(\phi)d\phi - A_f M_t \int_{-\infty}^{+\infty} h_{ft}(\phi)G(\phi)d\phi \right]$$
$$s.t. K_t = M_t \int_{-\infty}^{+\infty} k(\phi)G(\phi)d\phi$$
$$k_t(\phi) \geq 0, h_{mt}(\phi) \geq 0, h_{ft}(\phi) \geq 0 \;\; \forall t, \phi$$
$$K_t \geq 0, \lambda_{mt} \in [0,1], \lambda_{ft} \in [0,1], c_{jt} \geq 0 \;\; \forall j, t$$

where the maximization is with respect to $\{k_t(\phi), h_{mt}(\phi), h_{ft}(\phi), K_t, M_t\}$ for all t and ϕ. Denote by ϕ_t the Lagrange Multiplier in period t. The first order conditions are:

$$k_t(\phi): \phi_t = \beta^t u'(c_t)\phi F_k(k_t(\phi), h_{mt}(\phi), h_{ft}(\phi)) \tag{11}$$
$$h_{mt}(\phi): A_m = u'(c_t)\phi F_{h_m}(k_t(\phi), h_{mt}(\phi), h_{ft}(\phi)) \tag{12}$$
$$h_{ft}(\phi): A_f = u'(c_t)(\phi F_{h_f}(k_t(\phi), h_{mt}(\phi), h_{ft}(\phi)) - \nu) \tag{13}$$
$$K_t: \beta^t u'(c_t) = \beta^{t+1} u'(c_{t+1})\left((1-\delta_k)\right) + \phi_{t+1} \Rightarrow$$
$$u'(c_t) = \beta u'(c_{t+1})(\phi F_k(k_{t+1}(\phi), h_{mt+1}(\phi), h_{ft+1}(\phi)) + 1 - \delta_k) \tag{14}$$
$$M_t: \beta^t u'(c_t)\kappa = \beta^{t+1} u'(c_{t+1})\left(\tilde{Y}_{t+1} + (1-\delta_f)\kappa - \nu\tilde{\lambda}_{ft+1}\right) - \beta^{t+1} A_m \tilde{\lambda}_{mt+1} - \beta^{t+1} A_f \tilde{\lambda}_{ft+1} - \phi_{t+1}\tilde{K}_{t+1} \tag{15}$$

where a $\tilde{}$ denotes the variable divided by M_t.

Proposition 3 *The Pareto optimal allocation is the same as the undistorted competitive equilibrium allocation.*

Proof. Note first that, in the undistorted equilibrium,

$$r_t = \phi F_k(k_t(\phi), h_{mt}(\phi), h_{ft}(\phi))$$
$$w_{mt} = \phi F_{h_m}(k_t(\phi), h_{mt}(\phi), h_{ft}(\phi))$$
$$w_{ft} = \phi F_{h_f}(k_t(\phi), h_{mt}(\phi), h_{ft}(\phi))$$

Inserting these into equations (12), (13), and (14),

$$A_m = u'(c_t) w_{mt} \tag{16}$$
$$A_f = u'(c_t)(w_f - \nu) \tag{17}$$
$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = r_{t+1} + 1 - \delta_k \tag{18}$$

Finally, we show the free entry condition (2) is satisfied by the social planner. Use equations (16) and (17) into equation (15) to obtain:

$$\begin{aligned} u'(c_t)\kappa &= \beta u'(c_{t+1})\left(\tilde{Y}_{t+1} - (1-\delta_f)\kappa - \nu\tilde{\lambda}_{ft+1}\right) - \beta u'(c_{t+1}) w_{mt+1}\tilde{\lambda}_{mt+1} - \beta u'(c_{t+1})(w_{ft+1}-\nu)\tilde{\lambda}_{ft+1} \\ &\quad - \beta u'(c_{t+1})\phi F_k(k_{t+1}(\phi), h_{mt+1}(\phi), h_{ft+1}(\phi))\tilde{K}_{t+1} = \\ &= \beta u'(c_{t+1})\left(\tilde{Y}_{t+1} - w_{mt+1}\tilde{\lambda}_{mt+1} - w_{ft+1}\tilde{\lambda}_{ft+1} - r_{t+1}\tilde{K}_{t+1} + (1-\delta_f)\kappa\right) = \\ &= \beta u'(c_{t+1})\left(\int_{-\infty}^{+\infty} \pi_{t+1}(\phi)G(\phi)d\phi + (1-\delta_f)\kappa\right) \end{aligned} \tag{19}$$

Adding equation (18),

$$\kappa = \frac{1}{r_{t+1} + 1 - \delta_k}\left(\int_{-\infty}^{+\infty} \pi_{t+1}(\phi)G(\phi)d\phi + (1-\delta_f)\kappa\right) \tag{20}$$

Equations (16) through (20) characterize the undistorted competitive allocation. ■

6 Calibration

We set the utility function as logarithmic, that is, $u(c) = \ln(c)$.

We first describe the calibration of the production function. We use data of the manufacturing sector from the *Encuesta Nacional Industrial Anual* (ENIA), which is collected on a yearly basis by the *Instituto Nacional de Estadísticas* (INE). This database contains census information of all firms in the manufacturing sector in Chile with more than ten employees. Specifically, we use data of 4,554 firms in 2007.[8]

We then set the parameters of the production function to match the factors ratios observed in the data. The first order condition of the profit maximization of firms imply that for firms who are not subject to the policy

$$k/h_w = \left(\frac{w_f}{r}\right)^{\frac{1}{1-\sigma}}$$

and

$$h_m/h_w = \frac{w_f^{\frac{1}{1-\sigma}}\left(r^{\frac{\sigma}{\sigma-1}} + w_f^{\frac{\sigma}{\sigma-1}}\right)^{\frac{\sigma-\rho}{\sigma(1-\rho)}}}{w_m^{\frac{1}{1-\rho}}}$$

Thus, we use average factor ratios and factor prices to obtain the parameters σ and ρ from the above equation.

Caselli and Feyrer (2007) compute the marginal productivity of capital equal to 0.26 in Chile

[8] We exclude firms that report having no employees or capital and firms with gross production less than reported value added. These firms account for 9.6 percent of the sample in that year.

so we use this in our estimation. To obtain estimates of wages we use a gender wage gap of 75 percent and wages data from the manufacturing sector in Chile, obtaining an average female wage of CLP$431,667 and an average male wage of CLP$575,583 per month. We use firms hiring up to 17 women for this computation to avoid firms distorted by the regulation. The estimates obtained for σ and ρ are 0.23 and 0.05, respectively. The fact that $\sigma > \rho$ implies that women can be more easily substituted for capital than men. In other words, capital and men are more complementary than capital and women.

We then compute θ by using the identity that

$$\theta = \frac{\log(Y)}{\log\left(\left(\left(k^{\sigma} + h_f^{\sigma}\right)^{\rho/\sigma} + h_m^{\rho}\right)^{1/\rho}\right)}$$

using average value added as a proxy for production Y and the parameters σ and ρ previously estimated. Using that, we obtain a scale parameter of 0.6.[9]

Now we obtain β, which is related to the marginal productivity of capital. Recall that in steady state, $1/\beta = r + 1 - \delta_k$, where r is the marginal productivity of capital, so we set β to guarantee this.

The distribution for entering firms is a Pareto distribution. We assume this so that the upper tail of the distribution of firm sizes in the model is also a Pareto distribution in equilibrium. This is a very common parameterization due to the fact that it implies the calibration of a single parameter, and also that, in equilibrium, the firm size distribution for large firms is a Pareto distribution, which is a relatively good assumption. To see this, one can derive a measure of size as a function of productivity ϕ from the first order conditions to the firm problem. There are three natural candidates to use as proxy for firm size: output, male labor, and female labor. In all three cases, the size of a firm with productivity z is proportional to $z^{\frac{1}{1-\theta}}$.

The exogenous distribution of productivities is $G(\phi) = \alpha\phi^{-\alpha-1}$. The mass of men hired by an unconstrained firm with productivity ϕ is $h_m(z) = h_0 z^{\frac{1}{1-\theta}}$, where h_0 is a constant. We focus on the distribution of firm sizes for firms with more than h^* men. The productivity of a firm with more than h^* men is $\phi^* = \left(\frac{h^*}{h_0}\right)^{1-\theta}$. The fraction of firms with more than h^* men is given by

$$D_{h>h^*} = \int_{\hat{\phi}^*}^{\infty} F(\phi)d\phi = \int_{\phi^*}^{\infty} \alpha\phi^{-\alpha-1}d\phi = \phi^{*-\alpha} = \left(\frac{h^*}{h_0}\right)^{-(1-\theta)\alpha}$$

where $\hat{\phi}$ is the productivity of the firm hiring $\hat{h}_m$ men, that is, $\hat{\phi} = \left(h_m/\hat{h}_m\right)^{1-\theta}$.

[9] This procedure is based on Escobar et al. (2016).

Taking logs of this expression,

$$\log(D_{h>\hat{h}_m}) = constant - (1-\theta)\alpha \log h^*$$

Thus, we pin down α by using the slope of the firm size distribution measured by number of men in logs.

To calibrate the cost of child care ν we match the ratio of child-care costs to GDP in Chile. We do not have a direct measure of child-care cost, so we build it as follows. Between 2005 and 2007, an average of 234,261 children were born per year. The infant death rate is 0.0076, so around 232,481 of these survive. That is a total of almost 465,000 children under the age of 2 in 2007, who would be covered by the policy. On average, the cost of child care per children is $100,000 (Chilean Pesos) per month.

Both in the data and in the model, not all children go to a child-care facility. In the model, only those whose parent goes to work require child care. The calibration suggests that 20% of women work, so about this percentage of children need childcare. The GDP in Chile in 2007 was almost $86 trillions. Thus, total child-care cost relative to GDP is

$$\text{Child-care Cost} = \frac{0.20 \times 100,000 \times 12 \times 232,481 \times 2}{86,000,000,000,000} = 0.0013$$

Parameter	Target	Value
σ	Escobar et al. (2016)	0.23
ρ	Escobar et al. (2016)	0.05
θ	Escobar et al. (2016)	0.60
α	Firm size distribution by men	3.18
β	Marginal productivity of capital	0.85
ν	Child-care costs paid relative to GDP	0.0013
h^*	Share of firms hiring more 19 women	0.07
$\bar{h}$	Normalization	1.00
δ_k	Depreciation in National Accounts	0.09
δ_f	Firm death rate	0.12
A_m	Male labor participation rate	0.97
A_f	Male to female wage premium	0.72

Table 1. Calibration targets and parameters

We calibrate the parameter $\hat{h}$ by matching the share of firms that hire more women than the threshold allows. In the data, this threshold is 19 women. Almost 83% of the firms in our sample hire less than 19 women. Thus, we set $\hat{h}$ so that 83% of firms in the model hire less than $\hat{h}$ women.

We next calibrate the entry cost, κ. One option is to use the cost of starting a business published by the Doing Business reports of the World Bank. However, we found this number not to be very reliable. It computes the average cost of starting a business, and most businesses in Chile belong to the service sector. Our sample does not include those businesses, and it is reasonable to expect manufacturing businesses to have larger start up costs. Fortunately, κ does not have a large effect on our results. Thus, we set κ so that in our calibration, the measure of firms is equal to 1.

The depreciation rate δ_k is set to 0.09, as the Chilean National Accounts suggest. Firm death rate δ_f is 0.12, to match the death of firms in Chile.

We normalize the total time available for work per person $\bar{h}$ equal to 1. The parameters A_f and A_m are calibrated so that male labor participation is 71.62%, and the female wage is 75% as large as the male wage. The labor participation rate is from the CASEN ("Encuesta de Caracterización Socioeconómica Nacional"), a Chilean household survey carried out by the Chilean government. Our choices of parameter values are summarized in Table 1.

7 Results

Having elaborated the model and its calibration, we now turn to counterfactual exercises. We simulate the effects of changes in the existing policy along the transition to a new steady state. See Appendix B for details on how we compute the allocation. We then compare different statistics, paying particular attention to welfare (along the transition), labor participation rates (across steady state), and the wage premium (across steady states).

The first counterfactual exercise removes the distortion, thus achieving the optimal allocation when all individuals have equal weights in the social planner's problem. The second counterfactual subsidizes the childcare cost of women via lump sum taxes on households. The point of this exercise is to explore alternative policies that could improve the labor participation rates of women. The fourth counterfactual subsidizes childcare costs financed by a tax on profits. The point of this counterfactual is to explore more realistic policies that can increase female labor participation.

7.1 Misallocation of Resources

This section shows how the policy misallocates resources relative to the social planner's solution. The profits function, is, by construction, continuous in ϕ. Figure 1 shows how profits change with the technology parameter ϕ. The solid line shows the profit function in equilibrium. The dashed line shows what the profits would look like if firms did not have to pay the childcare costs, regardless of how many women they hire.

Notice that the dashed line is above the solid line. They overlap each other for $\phi < \phi_1$. For all other values, the dashed line lies above the solid line.

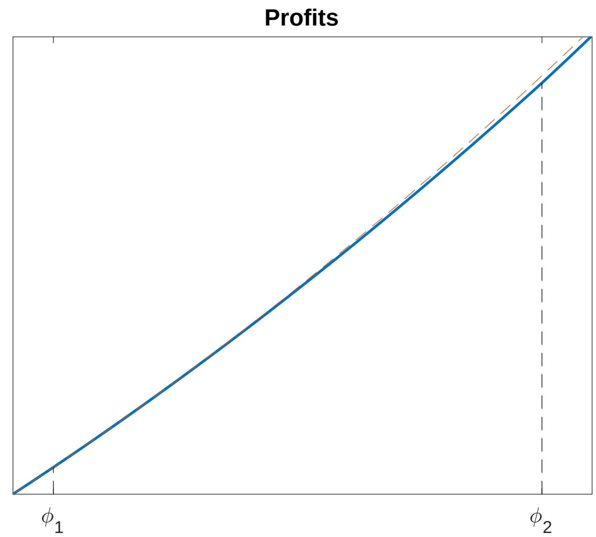

Figure 1. Profits as a function of ϕ

The effects of misallocation can be more clearly seen when focusing on output, the number of men, women, and capital hired by firms. Figure 2 shows these numbers. Once again, the dashed line represents the levels if there was no distortion.

These functions are no longer continuous. While they are continuous at the point ϕ_1, all series jump at ϕ_2. Clearly, the biggest jump is for women. This is because the hiring of women is constant between ϕ_1 and ϕ_2. After ϕ_2, the hiring of women also display the largest difference between the solid and dashed lines. This shows how female labor demand would increase absent the distortion, suggesting that the policy has its largest effect in the demand for female workers. What can be seen from this graph is that, according to the parameters we calibrated, it appears that most firms who try to avoid the childcare costs strongly reduce the number of women in their firm but only very slightly alter the amount of men workers and capital, suggesting that firms are not substituting extremely strongly women with other inputs but instead producing less. This is what can be seen from the fall in output observed at the same moment.

Figure 3 shows how different measures of productivity are affected by the distortion. Naturally, output per woman increases with the distortion. This is because by limiting the hiring of women, the ratio of men to women and capital to women increase, increasing output per woman. This is not so for output per man, or output per unit of capital. Productivity per person (men plus women) is a function of both output per man and output per woman. The figure shows that the distortion increases measured output per person, as expected due to the increase in capital.

As a direct consequence of the policy, firms demand less female labor. This is illustrated in Figure 4, where the demand for women relative to men and capital sharply drops for constrained firms. For unconstrained firms that hire more than 20 women, female labor demand is lower

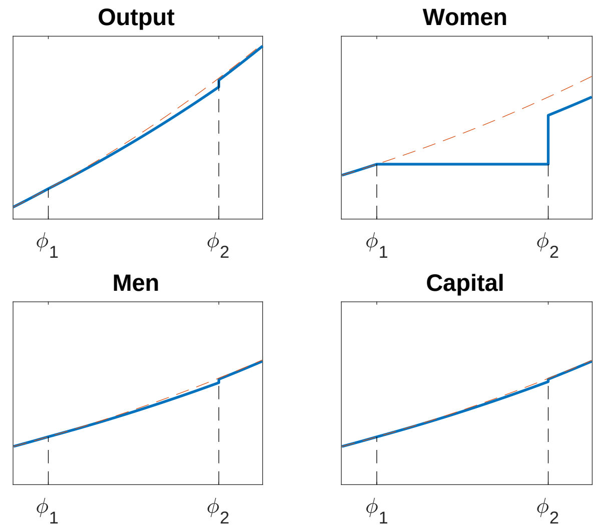

Figure 2. Output, Women, Men and Capital as a function of ϕ

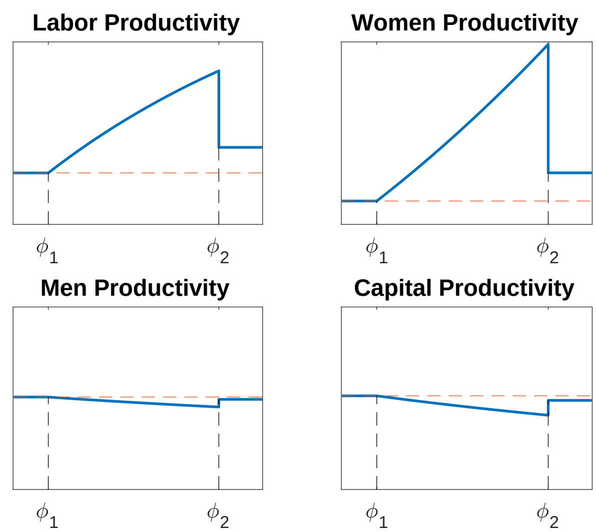

Figure 3. Different measures of productivity as a function of ϕ

relative to men and capital than for firms with less than 20 women because women are relatively costlier, the difference given by the cost of childcare.

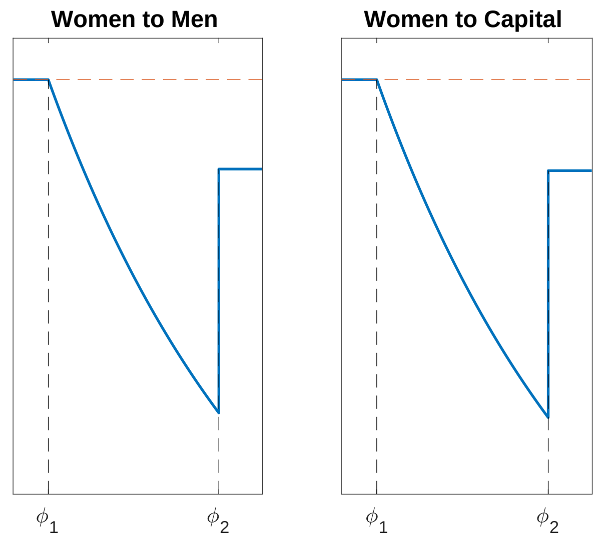

Figure 4. Ratio of women to men and women to capital employed

7.2 Counterfactual 1: Comparing the policy to first-best

In this section we evaluate the effect of the distortion on different outcomes by counterfactually eliminating the distortion. Tables 2 and 3 show the effects of eliminating the distortion on the outcomes studied.

Overall, the change in welfare is negligible, and it is fair to say that the policy has no bearing on aggregate welfare. This is mainly a result of how small the childcare costs are, of only a tenth of a percent of GDP. However, there are sizable effects on other dimensions, such as the distribution of welfare between men and women, the labor participation, and the wage premium.

Our results suggest that the distortive policy has a positive effect on women's welfare, albeit a very small one. A word of caution is required when interpreting our results. The assumption that there are complete markets means that any wage income, or production, is shared between men and women within household, and therefore an increase in hours worked will reduce the welfare of the gender increasing the hours worked directly, while the increase in consumption associated to that increase in hours worked will be spread evenly across genders.

In consumption equivalent units, eliminating the distortion would amount to reducing women's consumption by 0.05%, while increasing men's consumption by the same percentage. The reason is that with the removal of the distortion female labor increases (by 0.6% in steady state), and some of the extra consumption that women generate is consumed by men, so women are slightly worse off and men slightly better off.

Turning to other outcomes, we first analyze the effects on productivity. By eliminating the distortion GDP per capita would slightly drop, by 0.01%. This is a combination of women

Table 2. Effect of Different child care policies on welfare

	Current Policy	Lump-sum Tax Finances Childcare	Profit Tax Finances Childcare
Welfare for both along transition	1.0000	1.0000	1.0000
Welfare for men along transition	0.9999	1.0005	1.0006
Welfare for women along transition	1.0001	0.9995	0.9994
Welfare for both in steady state	1.0001	1.0005	0.9996
Welfare for men in steady state	1.0000	1.0011	1.0001
Welfare for women in steady state	1.0002	1.0000	0.9991
Consumption-equivalents men	0.9995	1.0021	1.0026
Consumption-equivalents women	1.0005	0.9978	0.9974

working more, and hence producing more, and men working and producing less. The effects on productivity would be less clear. This is because, as we show in Figure 3, the distortion has the effect of increasing women's productivity. Total productivity (output per person), decreases when eliminating the distortion, by 0.12%. This is driven by women's productivity, that drops by 0.59%. Men's productivity, on the other hand, increases by 0.01%, and so does capital productivity, by 0.02%. This is because the policy make firms use female work less than the social planner optimal, thus leading to increasing productivity by the fact that we have a production function that is concave.

The key of what happens lies in the labor market outcomes. We find that compared to first-best, women work less and men (and capital) work more. This is because while the policy makes women want to work more than before since they receive the child care benefits, firms want to avoid hiring them and the equilibrium outcome is one where women work less than in first-best. Wages are also negatively impacted by the policy. Eliminating the distortion increases female wages by almost 2%, while having almost no effect on men's wages. Thus, absent the childcare policy, the wage premium would be over 2% smaller (if we take wage premium to be 75% this imply that in absence of the policy female wages would be 76.3% of male wages).

7.3 Counterfactual 2: A Subsidy to Childcare Financed with Lump Sum Taxes to Households

Does providing child care necessarily lead to lower female participation as we found in the previous section? We know that the main reason behind the previous result was that the policy decreased the willingness of firms to hire women. What if we devise a program that would subsidized child care but without having that component? We now introduce subsidies, instead of mandates, to childcare to measure how these would impact female labor participation. We find this to be a very effective policy to increase female labor participation rates, with small welfare

Table 3. Effect of Different child care policies on productivity, prices and factors

	Current Policy	Lump-sum Tax Finances Childcare	Profit Tax Finances Childcare
GDP per capita	1.0001	1.0044	0.9991
Labor Productivity	1.0012	0.9979	0.9930
Men-Productivity	0.9999	1.0047	1.0000
Women-Productivity	1.0059	0.9743	0.9689
Capital-Productivity	0.9998	1.0007	1.0010
Consumption in steady state	0.9999	1.0044	0.9999
Men labor supply in steady state	1.0002	0.9997	0.9990
Women labor supply in steady state	0.9943	1.0309	1.0311
Capital in steady state	1.0003	1.0037	0.9980
Mass of firms in steady state	1.0007	1.0044	0.9957
Men wages	0.9999	1.0044	0.9999
Women wages	0.9828	0.9796	0.9752

consequences. Before presenting our results, we describe the problem faced under this situation.

The household's problem is

$$\max \sum_{t=0}^{\infty} \beta^t \left\{\lambda_{ft}(u(c_{fw,t}) - A_f\bar{h}) + (1-\lambda_{ft})u(c_{fn,t}) + \lambda_{mt}(u(c_{mw,t}) - A_m\bar{h}) + (1-\lambda_{mt})u(c_{mn,t})\right\}$$

$$s.t.$$

$$\sum_{t=0}^{\infty} p_t \left\{\lambda_{mt}c_{mw,t} + (1-\lambda_{mt})c_{mn,t} + \lambda_{ft}c_{fw,t} + (1-\lambda_{ft})c_{fn,t} + \right.$$

$$\left. K_{t+1} - (1-\delta_k)K_t + \kappa(M_{t+1} - (1-\delta_f)M_t) + \lambda_{ft}(1-s_t)\nu\right\} =$$

$$\sum_{t=0}^{\infty} p_t \left\{\lambda_{ft}w_{ft} + \lambda_{mt}w_{mt} + r_tK_t + \Pi_t - S_t\right\}$$

$$\lambda_{ft} \in [0,1], \lambda_{mt} \in [0,1], c_{fwt} \geq 0, c_{fnt} \geq 0, c_{mwt} \geq 0, c_{mnt} \geq 0, K_t \geq 0, K_0 \text{ given.} \quad \forall i,t$$

where s_t is the subsidy rate in period t, and S_t are the lump sum taxes required to finance these subsidies, that is, $S_t = s_t\nu\lambda_{ft}$. We set $s_t = 1$, so that women do not pay any childcare cost. The problem of incumbent firms does not change relative to the undistorted equilibrium, and neither do the market clearing or free entry conditions.

In particular, only one first order condition changes. In the undistorted economy, an equilibrium condition can be written as

$$u'(c_t)(w_{ft} - \nu) = A_{ft}$$

This condition now becomes

$$u'(c_t)w_{ft} = A_{ft}$$

Thus, given the same wages under both scenarios, women demand more consumption with the subsidy (c_t increases). Thus, hours worked need to increase, making sense from an intuitive point of view, since they do not need to worry about childcare. On the other hand, we can compute the relation between wages under both scenarios, having in mind that

$$u'(c_t)w_{mt} = A_{mt}$$

Without any distortion, the relation between wages is

$$\frac{w_{ft} - \nu}{A_f} = \frac{w_{mt}}{A_m}$$

and with childcare subsidy, the relation is

$$\frac{w_{ft}}{A_f} = \frac{w_{mt}}{A_m}$$

Thus, the subsidy tends to increase the wage premium, other things being equal.

Calibrating these effects, we present the results on welfare in the second column of Table 2. Overall, welfare for both men and women (the sum) would remain unaffected. However, this policy slightly shifts utility from women to men. It is noteworthy that the sum of utilities in steady state increases with the subsidy and the lump sum tax by about 0.04%, compared both to the undistorted case and the current situation. However, since the undistorted case is Pareto efficient, when computing the transition, the sum of the utilities cannot be larger than the undistorted case (it is in fact smaller, when enough decimal places are considered). The reason for these apparently contradictory results is that the subsidy, by driving more women to work, increases output relative to the undistorted case at the expense of having more women working. In steady state the additional consumption more than compensates the increase in labor costs. However, along the transition, the increase in consumption is more modest, and cannot offset the additional disutility of work.

As shown in Table 3, we find the subsidy to be extremely effective at increasing female labor participation, increasing it by 4%. It does however reduce female welfare, albeit by less than 0.04%, and increases the wage premium, since female wages would drop by a bit less than 1% and male wages would increase by 0.27%. This is because female labor supply is very incentivized through this channel and firms have no reason to decrease their hiring of women. The market clearing condition implies that wages must then fall. This generates the loss of utility from women.

7.4 Counterfactual 3: A Subsidy to Childcare Financed with Profit Taxes

Given the success of increasing female labor participation rates of a subsidy on childcare, we now explore a more realistic source of financing than lump sum taxes: a profit tax.

The household's problem is

$$\max \sum_{t=0}^{\infty} \beta^t \left\{ \lambda_{ft}(u(c_{fw,t}) - A_f \bar{h}) + (1-\lambda_{ft})u(c_{fn,t}) + \lambda_{mt}(u(c_{mw,t}) - A_m \bar{h}) + (1-\lambda_{mt})u(c_{mn,t}) \right\}$$

$$s.t.$$

$$\sum_{t=0}^{\infty} p_t \left\{ \lambda_{mt} c_{mw,t} + (1-\lambda_{mt})c_{mn,t} + \lambda_{ft} c_{fw,t} + (1-\lambda_{ft})c_{fn,t} + K_{t+1} - (1-\delta_k)K_t + \kappa(M_{t+1} - (1-\delta_f)M_t) + \lambda_{ft}(1-s_t)\nu \right\} = \sum_{t=0}^{\infty} p_t \left\{ \lambda_{ft} w_{ft} + \lambda_{mt} w_{mt} + r_t K_t + \Pi_t \right\}$$

$$\lambda_{ft} \in [0,1], \lambda_{mt} \in [0,1], c_{fwt} \geq 0, c_{fnt} \geq 0, c_{mwt} \geq 0, c_{mnt} \geq 0, K_t \geq 0, K_0 \text{ given.} \quad \forall i, t$$

where Π_t are now after tax profits.

While the first order conditions for firms do not change relative to the undistorted economy, the free entry condition is now:

$$\kappa = \tilde{q}_t((1-\tau_t)\int_{-\infty}^{+\infty} \pi_t(\theta) G(\phi) d\phi + (1-\delta_f)\kappa)$$

where τ_t is profit taxes. We set τ_t such that $\tau_t \int_{-\infty}^{+\infty} \pi_t(\theta)\mu_t(\phi)d\phi = \nu\lambda_{ft}$ in steady state. Note that we should impose this along the transition as well. However, this involves finding the solution of a non-linear equation at each point in time, which is very computational intensive. Thus, we only impose this in steady state. Given the small size of $\nu\lambda_{ft}$, we do not think the results would be considerably affected by finding the tax rate that balances the budget each period.

The effects of this policy are very similar to that of the lump sum tax, as can be seen in the third column of Tables 2 and 3, at least qualitatively. In general, however, the profit tax generates worse outcomes than the lump-sum tax. This is because firm decisions are now distorted, not in their input choices, as happened previously, but in how large they wish to be. A key difference is that, in steady state, there are less firms than with the lump sum tax. This is because the tax reduces the benefits from starting new firms, and consequently less firms are opened.

7.5 Robustness checks

Having shown that the policy, while distorting input choices, has limited impacts on welfare but substantial impacts on female employment in the economy, we now explore how robust these results appear to be. In particular, we explore the role of the parameters of our production function since the innovation of this paper is in part the use of 3 factors with distinct substitution elasticities. We find that we are unable to explore extremely different parameters without making our calibration fail to converge. However, we can explore substantial changes in parameters in any case. We first explore the impact of modifying σ by 20%. We find that the (very modest) impacts on welfare are unaffected by changes in σ. Nor are our more substantial impacts on female wages and labor supply.

Table 4. Sensitivity to the choice of σ in calculating the impact of child-care policies

	Current Policy		Lump-sum Tax		Profit Tax	
	$\sigma\Delta^{+}20\%$	$\sigma\Delta^{-}20\%$	$\sigma\Delta^{+}20\%$	$\sigma\Delta^{-}20\%$	$\sigma\Delta^{+}20\%$	$\sigma\Delta^{-}20\%$
Welfare for both along transition	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Welfare for men along transition	0.9999	0.9999	1.0004	1.0006	1.0005	1.0007
Welfare for women along transition	1.0001	1.0001	0.9997	0.9994	0.9996	0.9993
Welfare for both in steady state	1.0000	1.0001	1.0004	1.0007	0.9997	0.9995
Welfare for men in steady state	1.0000	0.9999	1.0008	1.0014	1.0001	1.0002
Welfare for women in steady state	1.0001	1.0002	1.0000	1.0000	0.9993	0.9988
Consumption-equivalents men	0.9996	0.9993	1.0015	1.0028	1.0018	1.0034
Consumption-equivalents women	1.0003	1.0006	0.9984	0.9971	0.9981	0.9966
GDP per capita	1.0001	1.0001	1.0030	1.0060	0.9993	0.9987
Labor Productivity	1.0009	1.0015	0.9981	0.9978	0.9948	0.9913
Men-Productivity	0.9999	0.9999	1.0032	1.0063	1.0000	1.0001
Women-Productivity	1.0061	1.0055	0.9719	0.9770	0.9679	0.9702
Capital-Productivity	0.9998	0.9998	1.0007	1.0007	1.0009	1.0010
Consumption in steady state	0.9999	0.9999	1.0030	1.0060	0.9999	0.9999
Men labor supply in steady state	1.0001	1.0002	0.9998	0.9996	0.9993	0.9987
Women labor supply in steady state	0.9940	0.9946	1.0320	1.0296	1.0325	1.0295
Capital in steady state	1.0002	1.0003	1.0023	1.0053	0.9985	0.9977
Mass of firms in steady state	1.0005	1.0010	1.0030	1.0060	0.9971	0.9940
Men wages	0.9999	0.9999	1.0030	1.0060	0.9999	0.9999
Women wages	0.9834	0.9825	0.9791	0.9807	0.9760	0.9748

We then turn to changes in the parameter ρ in Table 5. We find little reason to believe that the qualitative results were driven by the choice of that parameters. However, we do see that the labor supply of women would be particularly affected when ρ would be smaller than calibrated with an even lower level of wages. Alternative policies also see the wage falls more when ρ is smaller. This is because a fall in ρ implies a higher elasticity of substitution between women and other inputs.

Finally, we wish to understand if our results are limited because the policy is relatively small in scale or because of the potential substitution firms may be able to do. We thus explore what would be the impact of the program using the same calibrated parameters but making the cost

Table 5. Sensitivity to the choice of ρ in calculating the impact of child-care policies

	Current Policy		Lump-sum Tax		Profit Tax	
	$\rho\Delta^{+}20\%$	$\rho\Delta^{-}20\%$	$\rho\Delta^{+}20\%$	$\rho\Delta^{-}20\%$	$\rho\Delta^{+}20\%$	$\rho\Delta^{-}20\%$
Welfare for both along transition	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Welfare for men along transition	0.9997	1.0000	1.0012	1.0002	1.0015	1.0002
Welfare for women along transition	1.0002	1.0000	0.9989	0.9999	0.9987	0.9998
Welfare for both in steady state	1.0001	1.0000	1.0011	1.0002	0.9990	0.9999
Welfare for men in steady state	0.9999	1.0000	1.0026	1.0004	1.0004	1.0000
Welfare for women in steady state	1.0004	1.0001	0.9999	1.0000	0.9978	0.9997
Consumption-equivalents men	0.9992	0.9997	1.0031	1.0011	1.0036	1.0014
Consumption-equivalents women	1.0007	1.0002	0.9968	0.9989	0.9963	0.9986
GDP per capita	1.0001	1.0000	1.0059	1.0025	0.9987	0.9995
Labor Productivity	1.0015	1.0007	0.9976	0.9986	0.9911	0.9959
Men-Productivity	0.9999	0.9999	1.0064	1.0026	1.0001	1.0000
Women-Productivity	1.0055	1.0062	0.9776	0.9704	0.9705	0.9673
Capital-Productivity	0.9997	0.9999	1.0009	1.0004	1.0015	1.0005
Consumption in steady state	0.9999	1.0000	1.0059	1.0025	0.9999	1.0000
Men labor supply in steady state	1.0003	1.0001	0.9995	0.9998	0.9985	0.9995
Women labor supply in steady state	0.9946	0.9939	1.0290	1.0331	1.0291	1.0333
Capital in steady state	1.0004	1.0001	1.0049	1.0020	0.9972	0.9990
Mass of firms in steady state	1.0010	1.0003	1.0059	1.0025	0.9937	0.9978
Men wages	0.9999	1.0000	1.0059	1.0025	0.9999	1.0000
Women wages	0.9834	0.9822	0.9820	0.9768	0.9761	0.9743

of the program larger or smaller. The results are presented in Table 6. The results of this table suggests that the overall limited impact on aggregate welfare do not appear to be an artifice of the size of the program. The redistribution between men and women would be even more marked if the program had been larger in size. However, the impacts on labor productivity, female labor supply and wages are clearly amplified when we make the program larger. This suggests that if the program was to be made available for children who are older or if women were more active on the labor market when having young children, this program could generate very substantial decreases in labor market participation of women, while the alternative policies could increase very substantially the presence of women on the labor market.

Table 6. Sensitivity to the choice of ν in calculating the impact of child-care policies

	Current Policy		Lump-sum Tax		Profit Tax	
	$\nu\Delta^{-}50\%$	$\nu\Delta^{+}50\%$	$\nu\Delta^{-}50\%$	$\nu\Delta^{+}50\%$	$\nu\Delta^{-}50\%$	$\nu\Delta^{+}50\%$
Welfare for both along transition	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Welfare for men along transition	1.0000	0.9998	1.0002	1.0009	1.0002	1.0011
Welfare for women along transition	1.0000	1.0002	0.9999	0.9991	0.9998	0.9990
Welfare for both in steady state	1.0000	1.0001	1.0002	1.0010	0.9999	0.9992
Welfare for men in steady state	1.0000	0.9999	1.0003	1.0021	1.0000	1.0002
Welfare for women in steady state	1.0001	1.0003	1.0000	1.0000	0.9997	0.9982
Consumption-equivalents men	0.9998	0.9990	1.0007	1.0039	1.0008	1.0047
Consumption-equivalents women	1.0001	1.0008	0.9993	0.9959	0.9992	0.9951
GDP per capita	1.0000	1.0001	1.0013	1.0083	0.9997	0.9982
Labor Productivity	1.0004	1.0022	0.9993	0.9960	0.9979	0.9869
Men-Productivity	1.0000	0.9998	1.0014	1.0088	1.0000	0.9999
Women-Productivity	1.0018	1.0111	0.9920	0.9523	0.9903	0.9423
Capital-Productivity	0.9999	0.9996	1.0002	1.0014	1.0003	1.0019
Consumption in steady state	1.0000	0.9998	1.0014	1.0084	1.0000	0.9997
Men labor supply in steady state	1.0001	1.0003	0.9999	0.9995	0.9997	0.9982
Women labor supply in steady state	0.9983	0.9891	1.0094	1.0588	1.0095	1.0593
Capital in steady state	1.0001	1.0005	1.0011	1.0069	0.9994	0.9963
Mass of firms in steady state	1.0002	1.0012	1.0013	1.0083	0.9987	0.9917
Men wages	1.0000	0.9998	1.0014	1.0084	1.0000	0.9997
Women wages	0.9946	0.9682	0.9937	0.9621	0.9923	0.9538

Overall, we have explored alternative policies that are not included here. In particular, a policy where all firms, regardless of size, would have to pay for the policy would actually replicate first-best since it would eliminate the factor choice distortion and workers would pass the cost to the firm, without changing relative prices. A policy where firms would be subject to the law only when they crossed the threshold of 20 employees, regardless of their gender, would lead to a smaller decrease in the demand for women workers by firms although a potentially relevant impact on men's work. It is more complex computationally to calculate this but the intuition shown here would suggest that this policy would impact less negatively female labor supply. Female wages could thus potentially fall more than in the current context.

8 Conclusions

This paper studies the effects of a policy in Chile that forces large firms to pay childcare costs to women. This policy misallocates resources by driving firms to cut back on their hiring on women, and this can in principle have important aggregate effects on welfare and productivity. Indeed, we find that the aggregate effects are relatively modest. This is not surprising given the relative modest cost of childcare as a portion of GDP, of about 0.13%.

On the other hand, the distributive effects are larger: the policy actually shifts resources from women to men, by discouraging the hiring of women and substituting them for men and capital. Given that the aim of the policy was to help women, and encourage them to join the labor force, we find that the effects were opposite to the intended ones.

In the future, we hope to add heterogeneity in the type of women to see whether such policy may have distributional consequences between women within a given economy.

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A Proofs and derivations

The elasticity of substitution between capital and female labor can be found as followed. Consider how the ratio $\frac{k}{h_f}$ changes when r/w_f changes. From the first order conditions of the firm

$$\frac{k}{h_f} = \left(\frac{r}{w_f}\right)^{\frac{1}{\sigma-1}} \Rightarrow$$
$$\eta_{k,h_f} = \frac{d\log(k/h_f)}{d\log(r/w_f)} = \frac{1}{\sigma-1}$$

For men,

$$\frac{k}{h_m} = \left(\frac{r}{w_m}\right)^{\frac{1}{\sigma-1}} \left(1+(w_f/r)^{\sigma}\right)^{\frac{\sigma}{(\rho-1)(\rho-\sigma)}}$$

If we assume that the ratio of w_f/r does not change when r/w_m changes, then

$$\eta_{k,h_m} = \frac{d\log(k/h_m)}{d\log(r/w_m)} = \frac{1}{\rho-1}$$

A similar demonstration can be made for the elasticity of substitution between capital and male labor.

B Transition for Counterfactuals

We find the transition using a shooting algorithm. Shooting algorithms work by guessing a value for c_0. This pins down the entire transition using the first order conditions. Only the right guess for c_0 will make the system converge to the new steady state, so one can use deviations from the new steady state to improve the guess for c_0, until the new steady state is reached.

More specifically, the guess of c_0 determines a value for $K_1 + \kappa M_1$. Plugging into the Euler equation we can get the value of c_1, and from this, $K_2 + \kappa M_2$, and therefore the entire sequence of $\{c_t\}$. If at some point the series starts converging to something other than the new steady state, we update the guess of c_0 until the sequence converges to the new steady state.

Step 1. Guess a value for c_0

Step 2. From the market clearing equation, get $K_1 + M_1\kappa$

$$K_1 + M_1\kappa = Y_0 + (1-\delta_k)K_0 + (1-\delta_f)M_0\kappa - \nu H_{f0} - 2c_0$$

Step 3. Get K_1, M_1 and c_1 by solving a system of 9 equations and 9 unknowns:

$$\begin{aligned}
w_{f1} &= \frac{A_f}{A_m} w_{m1} + \nu \\
w_{m1} &= \frac{A_m}{u'(c_1)} \\
\frac{\pi_1}{\kappa} + (1-\delta_f) &= r_1 + 1 - \delta_k \\
\frac{u'(c_0)}{\beta u'(c_1)} &= r_1 + 1 - \delta_k \\
r_1 &= F_k(k_1, h_{m1}, h_{f1}) \\
w_{f1} &= F_{h_f}(k_1, h_{m1}, h_{f1}) \\
w_{m1} &= F_{h_m}(k_1, h_{m1}, h_{f1}) \\
\pi_1 &= \frac{Y_1 - r_1 K_1 - w_{m1}h_{m1} - w_{f1}h_{f1}}{M_1} \\
KM_1 &= K_1 + \kappa M_1
\end{aligned}$$

The first 4 equations come from the consumer FOC, the next 3 from the firms FOC, the 7th from the definition of average profits, and the last by definition. The unknowns are $w_{m1}, w_{f1}, r_1, c_1, K_1, M_1, \pi_1, h_{m1}, h_{f1}$.

Step 4. Knowing K_1, M_1 and c_1, go to Step 2. to find c_2, K_2 and M_2 and do this several times. Each time, check the value for K_{t+1}. Stop if

(a) $K_{t+1} > 1.1 \times K_t$. This means K_t is exploding. The choice for c_0 was too low. Choose a higher one.

(b) $K_{t+1} < 0.9 \times K_{ss}$. This means K_t is imploding. The choice for c_0 was too high. Choose a lower one.

Step 5. Stop when

$$(K_t - K_{ss})^2 + (K_{t+1} - K_{ss})^2$$

is small enough, where K_{ss} is the new steady state value. When this number is close to zero, we have reached the new steady state.